Math Notes

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§1. Spherical Coordinates Conversion

Example.

$$\rho = \sqrt{x^2 + y^2 + z^2} > 0$$
$$x = \rho \sin\theta \cos\theta$$
$$y = \rho \sin\theta \sin\theta$$
$$z = \rho \cos\theta$$

To use spherical coordinates conversion in triple integral

$$|J| = \rho^2 \sin\theta$$

Problem 1.1.
Calculate $\iiint_T e^{(x^2+y^2+z^2)^{\frac{3}{2}}} \, dv$ in terms of $T = \{(x, y, z) \mid x^2 + y^2 + z^2 \leq 1\}$

Problem 1.2.
Calculate volume of a sphere with radius a through triple integral.

$$V = \iiint_T dv$$

$$T = x^2 + y^2 + z^2 < a^2$$

§2. Stable Field

Example.
Does this field stable?

$$\vec{F} = \frac{2x}{1+x^2+y^2}\vec{i} + \frac{2y}{1+x^2+y^2}\vec{j}$$

§2.1. Potential Function

In 2D case $\vec{F} = P\vec{i} + Q\vec{j}$ are stable, it means $P_y = Q_x$. We want to find U so

$$F = \nabla U \Leftrightarrow (P, Q) = (U_x, U_y)$$

so $U_y = Q, U_x = P$ for example

$$U_x = P \Rightarrow U = \int P \, \mathrm{d}x + C(y).$$

Now we drived from U respect to y and equal to Q so we get $C(y)$.

But we use this formula because it's easier.

$$\begin{gathered} U = \int P \, \mathrm{d}x + \int Q^* \, \mathrm{d}y + C) \\ \text{or} \\ U = \int P^* \, \mathrm{d}x + \int Q \, \mathrm{d}y + C) \end{gathered} \tag{1}$$

Example.

$$F = e^y\vec{i} + xe^y\vec{j} \tag{2}$$

Example.
Determine the work done by F on an arbitrarily way from point $(0,0,0)$ to $(1, \frac{\pi}{2}, -1)$.

$$F = yz\cos(xy)\vec{i} + \sin(xy)\vec{k} + xz\cos(xy)\vec{j} \tag{3}$$

Answer:

$$\begin{gathered} c := \begin{cases} x = 0 + t \\ y = 0 + \frac{\pi}{2}t, \\ z = 0 - t \end{cases} \\ 0 \le t \le 1 \end{gathered} \tag{4}$$

Theorem 2.1.1 (Green).
Assume C is a positively oriented, piecewise smooth, simple closed curve, Also P and Q have continuous partial derivatives over D.

$$\oint_C F.dr = \oint_C Pdx + Qdy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA \tag{5}$$

Example.
Proof the Green's theorem Theorem 2.1.1 for the following field, assume C is a circle with equation $x^2 + y^2 = 1$ positively oriented.

$$F = (2x + y)\vec{i} + 2y\vec{j} \tag{6}$$